
The generalized Gelfand–Graev characters of $GL_n(\mathbb{F}_q)$

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Abstract

Introduced by Kawanaka in order to find the unipotent representations of finite groups of Lie type, generalized Gelfand–Graev characters have remained somewhat mysterious. Even in the case of the finite general linear groups, the combinatorics of their decompositions has not been worked out. This paper re-interprets Kawanaka’s definition in type A in a way that gives far more flexibility in computations. We use these alternate constructions to show how to obtain generalized Gelfand–Graev representations directly from the maximal unipotent subgroups. We also explicitly decompose the corresponding generalized Gelfand–Graev characters in terms of unipotent representations, thereby recovering the Kostka–Foulkes polynomials as multiplicities.

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