Diagonally and antidiagonally symmetric alternating sign matrices of odd order

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Abstract

We study the enumeration of diagonally and antidiagonally symmetric alternating sign matrices (DAS-ASMs) of fixed odd order by introducing a case of the six-vertex model whose configurations are in bijection with such matrices. The model involves a grid graph on a triangle, with bulk and boundary weights which satisfy the Yang–Baxter and reflection equations. We obtain a general expression for the partition function of this model as a sum of two determinantal terms, and show that at a certain point each of these terms reduces to a Schur function. We are then able to prove a conjecture of Robbins from the mid 1980’s that the total number of $(2n + 1) \times (2n + 1)$ DASASMs is $\prod n (3i)!$, and a conjecture of Stroganov from 2008 that the ratio between the numbers of $(2n+1) \times (2n+1)$ DASASMs with central entry -1 and 1 is $n/(n + 1)$. Among the several product formulæ for the enumeration of symmetric alternating sign matrices which were conjectured in the 1980’s, that for odd-order DASASMs is the last to have been proved.

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