
Schur polynomials and matrix positivity preservers

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Résumé

A classical result by Schoenberg (1942) identifies all real-valued functions that preserve positive semidefiniteness (psd) when applied entrywise to matrices of arbitrary dimension. Schoenberg's work has continued to attract significant interest, including renewed recent attention due to applications in high-dimensional statistics. However, despite a great deal of effort in the area, an effective characterization of entrywise functions preserving positivity in a fixed dimension remains elusive to date. As a first step, we characterize new classes of polynomials preserving positivity in fixed dimension. The proof of our main result is representation theoretic, and employs Schur polynomials. An alternate, variational approach also leads to several interesting consequences including (a) a hitherto unexplored Schubert cell-type stratification of the cone of psd matrices, (b) new connections between generalized Rayleigh quotients of Hadamard powers and Schur polynomials, and (c) a description of the joint kernels of Hadamard powers.

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