Minimal factorizations of a cycle: a multivariate generating function

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Abstract

It is known that the number of minimal factorizations of the long cycle in the symmetric group into a product of k cycles of given lengths has a very simple formula: it is nk–1 where n is the rank of the underlying symmetric group and k is the number of factors. In particular, this is nn–2 for transposition factorizations. The goal of this work is to prove a multivariate generalization of this result. As a byproduct, we get a multivariate analog of Postnikov's hook length formula for trees, and a refined enumeration of final chains of noncrossing partitions.

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