Parabolic double cosets in Coxeter groups

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Résumé

Parabolic subgroups WI of Coxeter systems (W,S) and their ordinary and double cosets W/WI and WI/WJ appear in many contexts in combinatorics and Lie theory, including the geometry and topology of generalized flag varieties and the symmetry groups of regular polytopes. The set of ordinary cosets wWI , for I $\subseteq$ S, forms the Coxeter complex of W , and is well-studied. In this extended abstract, we look at a less studied object: the set of all double cosets WIwWJ for I,J $\subseteq$ S. Each double coset can be presented by many different triples (I, w, J). We describe what we call the lex-minimal presentation and prove that there exists a unique such choice for each double coset. Lex-minimal presentations can be enumerated via a finite automaton depending on the Coxeter graph for (W, S). In particular, we present a formula for the number of parabolic double cosets with a fixed minimal element when W is the symmetric group Sn. In that case, parabolic subgroups are also known as Young subgroups. Our formula is almost always linear time computable in n, and the formula can be generalized to any Coxeter group.

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