Noncrossing partitions, toggles, and homomesy

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Abstract

We introduce n(n – 1)/2 natural involutions ("toggles") on the set S of noncrossing partitions π of size n, along with certain composite operations obtained by composing these involutions. We show that for many operations T of this kind, a surprisingly large family of functions f on S (including the function that sends π to the number of blocks of π) exhibits the homomesy phenomenon: the average of f over the elements of a T -orbit is the same for all T -orbits. Our methods apply more broadly to toggle operations on independent sets of certain graphs.

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