
Oriented Flip Graphs and Noncrossing Tree Partitions

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Abstract

Given a tree embedded in a disk, we define two lattices - the oriented flip graph of noncrossing arcs and the lattice of noncrossing tree partitions. When the interior vertices of the tree have degree 3, the oriented flip graph is equivalent to the oriented exchange graph of a type A cluster algebra. Our main result is an isomorphism between the shard intersection order of the oriented flip graph and the lattice of noncrossing tree partitions. As a consequence, we deduce a simple characterization of c-matrices of type A cluster algebras.

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