Defining amplituhedra and Grassmann polytopes

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Abstract

The totally nonnegative Grassmannian Gr≥0 k,n is the set of k-dimensional subspaces V of Rn whose nonzero Plucker coordinates all have the same sign. In their study of scattering amplitudes in N = 4 supersym- metric Yang-Mills theory, Arkani-Hamed and Trnka (2013) considered the image (called an amplituhedron) of Gr≥0 k,n under a linear map Z : Rn → Rr, where k ≤ r and the r × r minors of Z are all positive. One reason they required this positivity condition is to ensure that the map Gr≥0 k,n → Grk,r induced by Z is well defined, i.e. it takes everynelement of Gr≥0 k,n to a k-dimensional subspace of Rr. Lam (2015) gave a sufficient condition for the induced map Gr≥0 k,n → Grk,r to be well defined, in which case he called the image a Grassmann polytope. (In the case k = 1, Grassmann polytopes are just polytopes, and amplituhedra are cyclic polytopes.) We give a necessary and sufficient condition for the induced map Gr≥0 k,n → Grk,r to be well defined, in terms of sign variation. Using previous work we presented at FPSAC 2015, we obtain an equivalent condition in terms of the r × r minors of Z (assuming Z has rank r).

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