

Counting Maps on Surfaces

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I will talk about maps, which are graphs embedded on surfaces. The enumeration of maps was initiated in the 1950's by Tutte, who discovered that planar maps (when the surface is the sphere) are counted by beautiful formulas. This was just the beginning of a story that is still developing today and has connections to almost every part of combinatorics.

The question of understanding how the enumerative properties of maps depend on the genus $g \geq 0$ of the underlying surface is especially interesting. Very strong results can be proved, coming from a variety of techniques, from representation theory of the symmetric group, to generatingfunctionology, integrable systems and tau-functions, or bijective combinatorics. However we still lack a general theory encapsulating all these results together. The look for a unification raises many questions and challenges for each of the tools involved.

In this talk I will show some of these results and some of these connections.